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ASSESSMENT OF ROBUST CONTROL ON DAMAGE GROWTH

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Abstract

The past few years have seen significant advances in the fields of structural health monitoring and damage prognosis, particularly in the areas of damage detection and localization. Because of these advances, it is conceivable, for the first time, to design a structure with the ability to detect damage and take corrective action to minimize its effects and slow its progression. This paper evaluates the potential for this damage mitigating control to allow mechanical, aerospace, and civil systems to monitor structural health internally, diagnose any damage, and then take action to intelligently mitigate the effect and progression of the damage. Specifically, damaged and undamaged cantilever beams are studied and a robust controller is used to damp vibration. The trade off between damping performance and increasing the strain at the damaged area, which is correlated with damage growth, is then investigated.

1. Introduction

A structure that has the ability to compensate for damage in such a manner that it slows damage progression has significant implications from both economic and safety perspectives. For instance, the tragic break-up of the space shuttle Columbia has been attributed to tiles on the left wing that were damaged during takeoff. During reentry, damage to those tiles increased the drag on that wing, which caused the flight control system to compensate while maintaining the optimal attitude of reentry. However, the increased efforts exerted by the control system to counter the effects of the increased drag may have inadvertently sped the progression of the space shuttle break up.

A control design that balances maintaining the system performance and preventing or slowing additional damage represents a recent development in the field of control design. An example of this strategy comes from what naturally occurs in the human body. If a person’s leg is injured, that person will strike a balance between performance and further damage prevention by limping. This type of control is similar to a life extending control.

The goal of life extending controllers (LECs) is to balance the trade-off between performance and structural durability [1]. Life extending controllers are currently being used in the aeronautics and astronautics industries in reusable rocket engines and airframe structures, including the Space Shuttle Main Engine, the F-15 and the F-18 [2]. In one study performed on an aircraft, tests showed a 40% increase in fatigue life could be achieved without significant performance loss when using an LEC [3]. Another test showed that the fatigue life of a mass-beam testbed could be up to three and a half times that of a system that does not have LEC, without significant sacrifice in performance [1]. LEC designs require knowledge of the areas of the structure most sensitive to damage, which is then used in designing the controller to monitor these areas [1]. However, little work has been done to assess how a controller should be modified once damage is present, particularly if it appears in an unexpected area of the structure, as could reasonably be expected in, for example, fighter aircraft during combat missions.

This study investigates the issue of control design when damage is present. Specifically, if a robust controller is designed for a plant, and that plant then incurs damage at a particular location, this study investigates the following questions: 1) Would a different original controller decrease the resultant fatigue loading and therefore slow the progression of damage? 2) If so, how does the performance of the damaged system under this new controller compare with the performance of the damaged system under the original controller? These questions are addressed through a simple analytical study, as well as through a simple experimental study.

Section 2 describes the analytical model used in this study. Section 3 discusses the experimentation development, setup and analysis. Section 4 presents the conclusions of the studies. Sections 5 and 6 present acknowledgements and references respectively.

2. Analytical Study

Problem Description

To investigate the hypothesized fatigue/performance tradeoff, a simple numerical example was considered. A series of six masses, coupled together with springs and dampers was analyzed, as shown in Figure 1.

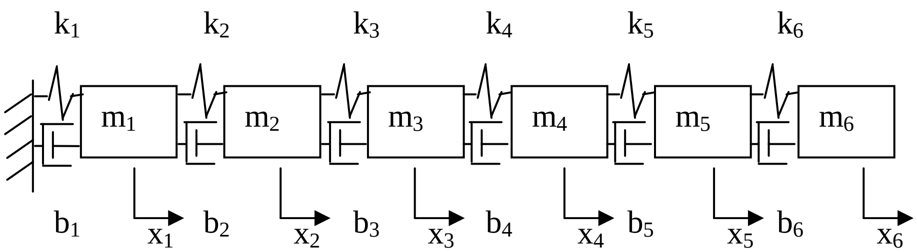


Figure 1. Spring, mass, damper system

In Figure 1, x_1 denotes the position of the first mass, k_1 denotes the first spring, b_1 denotes the first damper, and so on. The masses all have a value of 1.0, the springs all have a value of 10.0, and the dampers all have a value of 0.1. The equations describing this system are given by

$$M\ddot{x} + B\dot{x} + Kx = B_1 f_c + B_2 f_d$$

where *M*, *B*, and *K* are the familiar mass stiffness and damping matrices, f_c and f_d are the control and disturbance forces, respectively, and B_1 and B_2 are column vectors identifying which masses these control and disturbance forces act on. This system may then be easily rewritten in the state space representation

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}B \end{bmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + M^{-1}B_1 f_c + M^{-1}B_2 f_d \tag{1}$$

To assess what effect “damage” had on “fatigue” and “performance”, and what roll the “controller” played, “damage”, “fatigue”, “performance”, and “controller” must all be defined. The system is acted upon by a random, external disturbance force at one of the masses. The performance measure of interest is the RMS value of the vibration of mass 6, as caused by the disturbance. The performance objective of the controller is to minimize this RMS value. The controller itself produced a force which acts on one of the masses. Damage is simulated by decreasing the stiffness of one of the springs by some percentage. The locations of the disturbance and control forces and the location and severity of the simulated damage are variables under consideration. The fatigue loading, *F*, of a damaged spring is estimated as

$$F \approx \int_0^{NF} \omega \cdot U(\omega) U(\omega)^* \, d\omega$$

where NF is the Nyquist frequency. $U(\omega)$ is the Fourier transform of the deflection time history of the spring. While more complicated and more accurate techniques for estimating fatigue damage exist, this simple generalization of Miner’s rule is viewed as adequate because only relative changes in fatigue loading are of interest for the purposes of this paper.

For the purposes of this simple numerical model, the linear quadratic regulator (LQR) [4] was employed. The linear quadratic regulator is a well known full-state feedback controller that provides guaranteed robustness. The control is given by

$$f_c = -\mathbf{Kx}$$

where **x** is the state vector of positions and velocities of the masses. The state feedback gain matrix, **K**, is found through the solution of the constrained minimization of the cost function

$$J = \int_0^\infty \mathbf{x}^T \mathbf{Q} \mathbf{x} + f_c \cdot R \cdot f_c \, dt$$

subject to Eq. 1. Without loss of generality, R is set to 1. Then, by allowing $\mathbf{Q}$ to vary from $\mathbf{Q} << 1$ to $\mathbf{Q} >> 1$, the performance can be varied from a system in which suppressing the effects of the disturbance is not very important to a system in which suppressing the effects of the disturbance is very important.

Analysis Procedure and Results

The procedure for investigating the relationship between control and damage growth is as follows.
1) Select a control location.
2) Select a disturbance location. The disturbance signal is white noise.
3) Select a nominal **Q** and design a LQR. Simulate the closed loop system, and calculate the RMS value of the displacement of mass 6, as caused by the disturbance.
4) Select a damage level and location
5) Simulate the damaged closed loop system, and calculate the RMS value of the displacement of mass 6. Also, calculate the fatigue loading estimate for the “damaged” spring, using Eq. 2.
6) Repeat step 5 with several different controllers, as obtained from step 3, using different values for **Q**, to develop the relationship between performance and damage accumulation.

The above procedure was carried out for all six control, disturbance, and damage locations and two damage levels (0 and 0.5) for a total of 432 analyses. Each analysis examined 40 different controllers, obtained from 40 different **Q** matrices. The **Q** matrix used for each controller was the identity matrix multiplied by a scale factor that varied from 0.01 to 100. The result for a given analysis is a curve relating fatigue at the location of interest to the attenuation of the vibration of mass 6, which was the performance objective. There are two general forms that this curve can take. The first, shown in Figure 2, shows the possibility that, at least over the range **Q** matrices considered, the minimum fatigue corresponds to the best attenuation (lower is better). Thus, in cases such as this, there is no fatigue life penalty to be paid for maximizing performance.

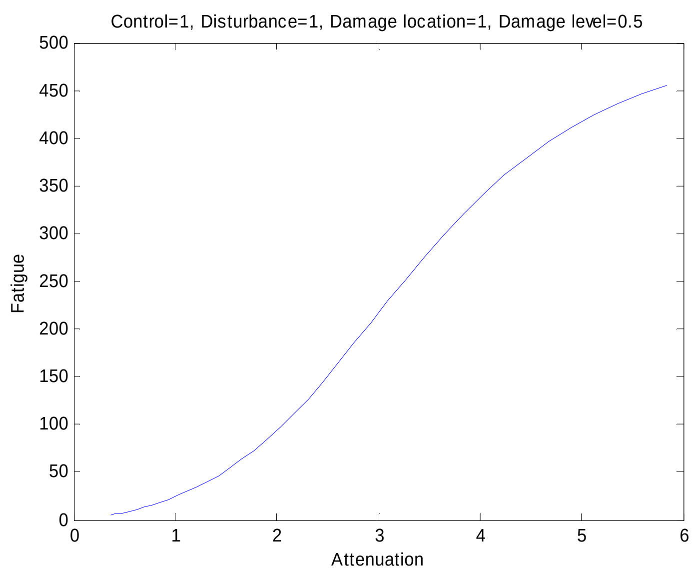


Figure 2. Fatigue/Attenuation Curve 1

The second form of this attenuation/fatigue curve is shown in Figure 3. In cases such as this there is a very strong tradeoff between performance and fatigue life. That is, over the range of Q matrices considered the controller that gives the best performance is not the same controller that gives the minimum fatigue.

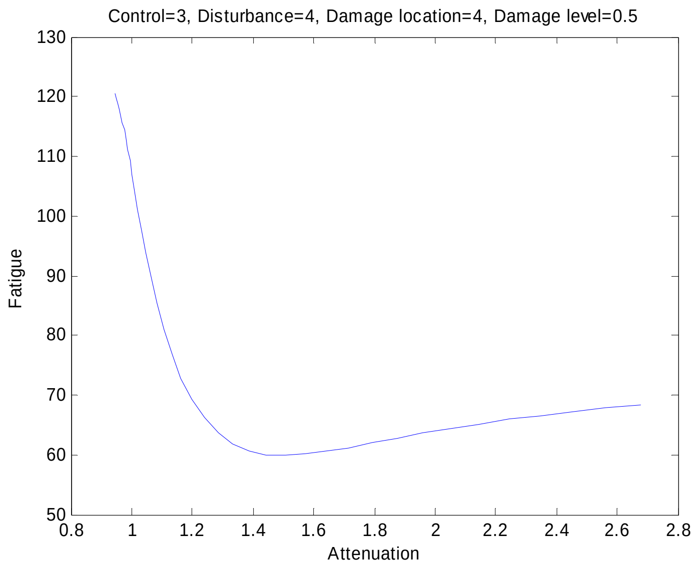


Figure 3. Fatigue/Attenuation Curve 2

In order to quantify the potential tradeoff between performance and fatigue, consider Figure 4. Under the assumption that the controller is originally designed for maximum performance, then the following metric for reduction of fatigue loading may be defined.

$$\%\, reduction\ in\ fatigue = \frac{fatigue\big|_{best\ attenuation} - \min\left(fatigue\right)}{fatigue\big|_{best\ attenuation}} \cdot 100$$

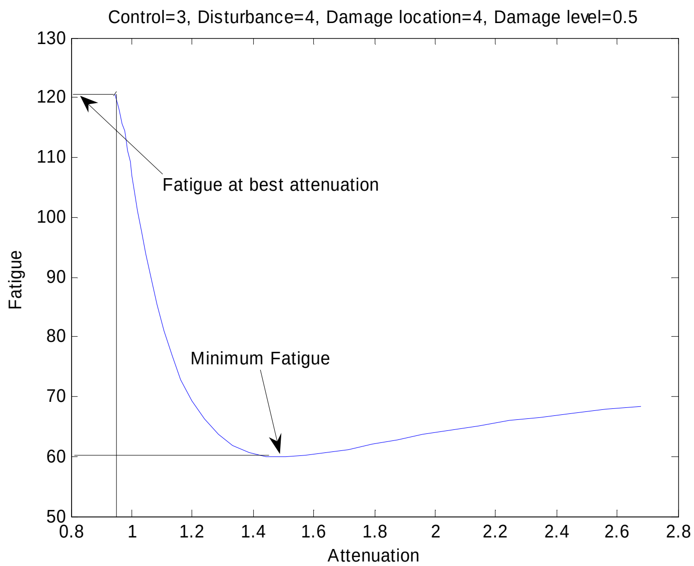


Figure 4. Key Features of Fatigue/Attenuation Curve

This fatigue reduction metric was calculated for all analyses. The results for the damage level 0 case are shown in Figure 5. The results for the damage level 0.5 case are shown in Figure 6.

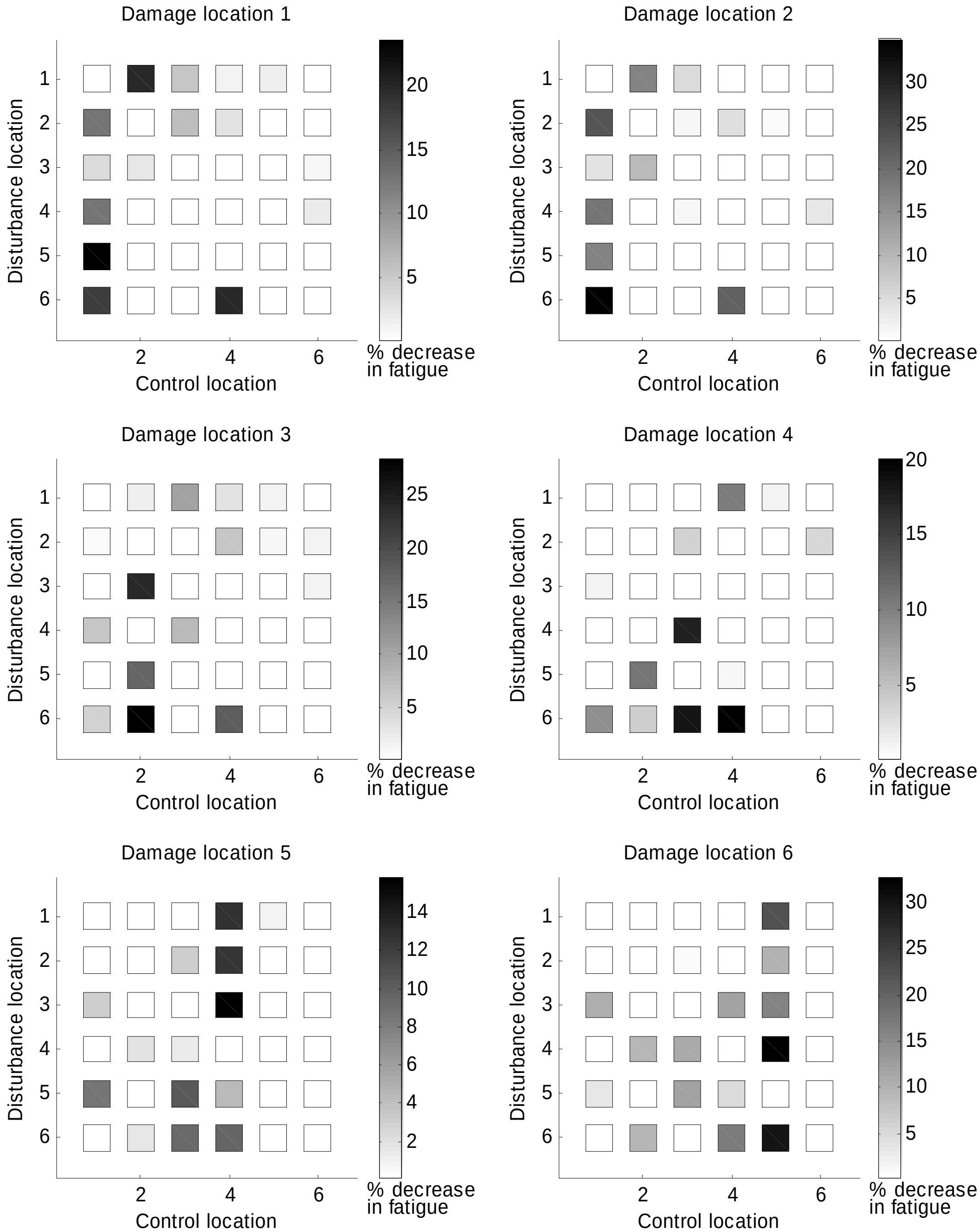


Figure 5. Fatigue Reduction Summary – Damage Level 0.0

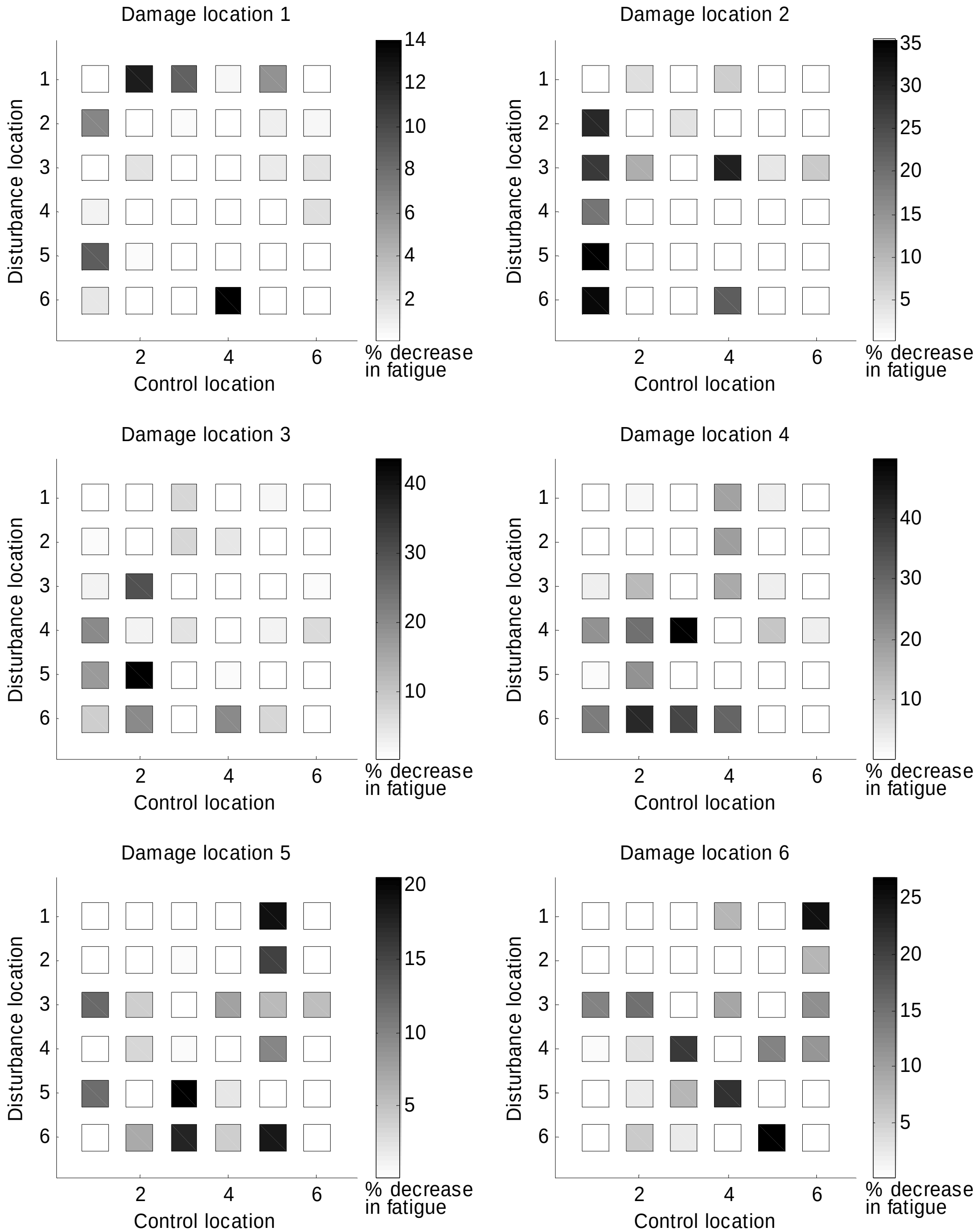


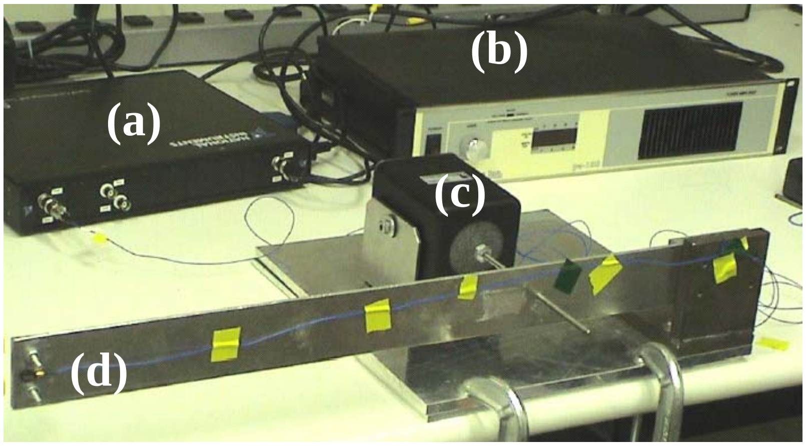
Figure 6. Fatigue Reduction Summary – Damage Level 0.5

While Figures 5 and 6 are information dense, one can make a few observations. The first is that there seem to be almost as many cases where a performance/fatigue tradeoff exists as cases where no such tradeoff exists. This implies, not surprisingly, that it would be difficult if not impossible to design a single controller that could gracefully handle disturbances and damage at locations that are not known *a priori*. Another significant observation is that no performance/fatigue tradeoff was observed for any damage level or any damage location when the control and disturbance were collocated. Though difficult to discern visually from Figures 5 and 6, in all cases where the control and disturbance were collocated, the controller that gave the minimum fatigue loading also gave the best performance. If this observation were found to be true for a large number of applications, the implications would be significant. By designing a system to have collocated control and disturbances, one may be able to significantly reduce fatigue and damage concerns, while maintaining excellent performance.

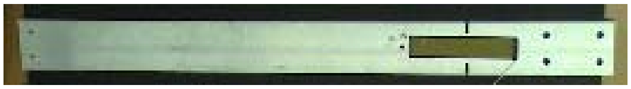
3. Experimental Study

Experimental Description

For the experimental portion of the study, a cantilever beam was studied. A photo of the experimental setup is shown in Figure 7.



(a)



(b)

Figure 7: a) Experimental Setup, b) Damaged Beam With Piezo Patch Mounted

The cantilever beam measures 24 x 2 x 0.125 inches and is made from 6061 aluminum. The damaged beam has 0.75 inch notches cut on both sides, at the point halfway between the shaker attachment and the cantilevered end. The beam.is instrumented with a PCB 352A24 accelerometer (item (d) in Figure 7(a)) to measure the acceleration of the free end of the beam. This acceleration signal is conditioned with a National Instruments SC2345 signal conditioner (item (a) in Figure 7(a)) using an ICP module. A PC running the Mathworks XPC Target and containing a National Instruments PCI-6052E I/O card reads the conditioned signal, implements a digital controller, and outputs a control signal to the Labworks PA-138 amplifier which then sends the amplified control signal to the Labworks ET-132-2 electromechanical shaker. A piezo patch mounted near the cantilevered end (as shown in Figure 7(b)) provides the strain measurement used to calculate the estimate of fatigue loading. The disturbance is artificially generated band limited noise (< 20 Hz) which is added to the control signal. Thus, the control and disturbance are collocated.

Because only output feedback, rather than state feedback, is available, a LQR could no longer be used for control. Instead, a simple low pass Butterworth filter was used for control, with the performance altering parameter being the gain. A block diagram of the system is shown in Figure 8. The discrete transfer function for the controller is given by

$$C(z) = 10^{-12} \cdot \frac{0.0030z^7 + 0.0207z^6 + 0.0620z^5 + 0.1034z^4 + 0.1034z^3 + 0.0620z^2 + 0.0207z + 0.003}{z^7 - 6.92z^6 + 20.55z^5 - 33.88z^4 + 33.51z^3 - 19.89z^2 + 6.559z - 0.9270} \tag{2}$$

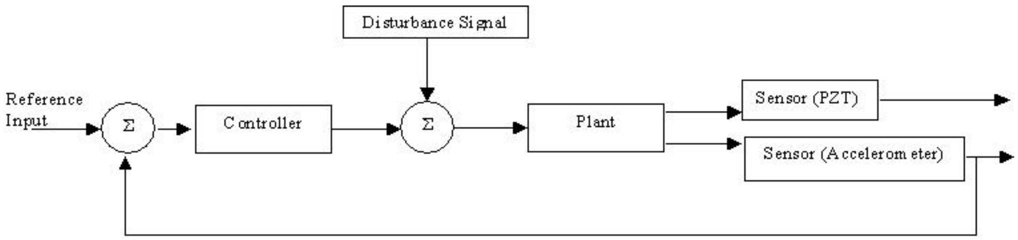


Figure 8. System Block Diagram

Experimental Procedure and Results

Two damage cases were considered. The first was an undamaged beam, as shown if Figure 7(a). A piezo patch was attached near the cantilevered to provide the fatigue loading estimate at that location. The second damage case was the beam shown in Figure 7(b), which has a notch that is approximately half the width of the beam. For each damage case the controller given in (2) was applied with gains ranging from 0.1 to 20. For each gain, the RMS amplitude of the acceleration at the free end was recorded, as was the strain at the damage location. The attenuation/fatigue curve for the undamaged beam is shown in Figure 9. The attenuation/fatigue curve for the damaged beam is shown in Figure 10.

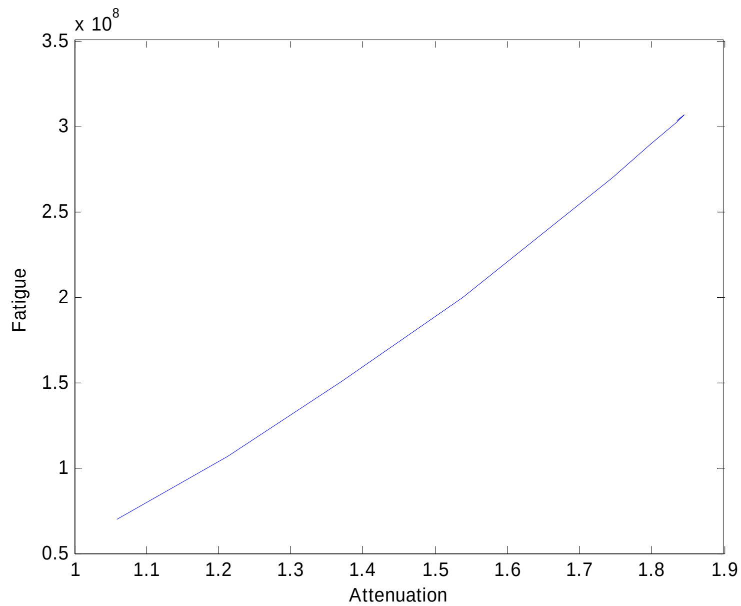


Figure 9. Fatigue/Attenuation Curve for Undamaged Beam

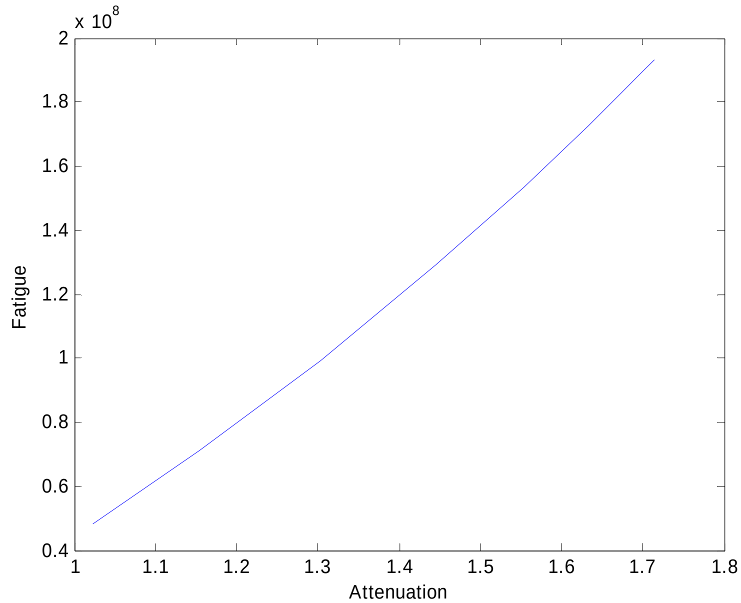


Figure 10. Fatigue/Attenuation Curve for Damaged Beam

Clearly, there is no performance/fatigue life trade off for either damage case, as the controller that produces the minimum fatigue loading at the damage location also gives the best performance. This result in not surprising in view of the observations from the analytical study regarding collocated control and disturbance.

4. Conclusions

Motivated by real world applications, a study was conducted to investigate the relationship between control system performance and structural fatigue loading. While previous work has addressed minimizing fatigue loading in certain likely failure regions which are known *a priori*, this study addressed the effect the control system has on fatigue loadings at arbitrary locations not known *a priori*. The results indicated that even for a system as simple as a series of masses connected with springs and dampers, there are some combinations of control location, disturbance location and damage location that require a careful optimization of the controller, while there are other locations that do not require a careful optimization. One particularly interesting result was the observation that, at least for this simple series-of-masses example, no careful optimization of the controller was required when the control and disturbance were collocated. That is, the controller that gave the best performance also gave the minimum fatigue.

Future work will investigate what effect control architecture (e.g., PID, LQR, fuzzy, etc.) has on this performance/fatigue relationships observed in this study. Additionally, experimental studies will be conducted in which the control and disturbance are not collocated.

5. Acknowledgements

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6. References

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